國立交通大學試題紙

一百學年度第一次
博士班資格考

科目：演算法(A)　　日期： 101 年 1 月 17 日　

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1. Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n \leqq 2$. Make your bounds as tight as possible, and justify your answers.
   (a). (3%) $T(n)=T(9n/10)+n$
   (b). (3%) $T(n)=T(\sqrt{n})+1$

2. Let $P_1, P_2, \ldots, P_n$ be n programs to be loaded to the main memory for execution. Program P_i requires s_i mega-bytes of memory space and the total size of the main memory is D mega-bytes, where $D < \sum_{i=1}^{n} s_i$. We want to maximize the number of programs loaded in the main memory.
   (a). (6%) Give an optimal greedy algorithm to do this.
   (b). (6%) Prove that the algorithm is correct.

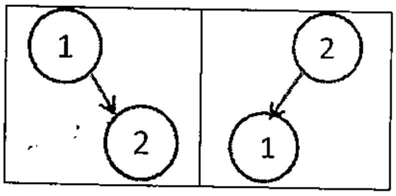

Figure 1. Two differently shaped binary search trees for storing 2 distinct items

3. When storing n distinct items in a binary search tree, the final shape of the binary search tree will depend on the insertion order of the n items and is not unique. Let H be the set of all valid binary search trees for storing n distinct items. As an example, for n=2, H will contain two differently shaped trees, which are shown in Figure 1.

   Dr. Tony needs a routine to randomly pick a binary search tree from H following the **uniform distribution** (i.e. each tree in H has an equal probability of being picked). He finally comes up with the following routine:

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```
PICKRANDOMTREE (A[1..n])
  // Input: array A containing n distinct items
  // Output: Among all valid binary search trees that store the n distinct items, select
           one of them following the uniform distribution and returns that binary
           search tree as T

  // RAND(n): return an random integer in the range [1,n].   All integers in the range
           [1,n] are equally likely.
  // SWAP(A[i], A[j]): swap elements i and j in array A
  // TREE-INSERT(T, z): insert item z into binary search tree T (no rebalancing).

  T←∅
  for i=n downto 1
    SWAP(A[i], A[rand(i)])

  for i=1 to n
    TREE-INSERT(T, A[i])

  return T
```

(a). (5%) What is the running time of Dr. Tony's PICKRANDOMTREE routine? Please describe it in asymptotic notation. You need to justify your answers.

(b). (6%) Please prove (or disprove) the algorithmic correctness of Dr. Tony's PICKRANDOMTREE routine.

4. Please answer the following short questions on network flow. Your answer should be as brief as possible and emphasize conceptual clarity over notational rigor.
   (a). (2%) What is a cut of a flow network?
   (b). (2%) What is the flow across a cut?
   (c). (2%) What is the capacity of a cut?
   (d). (2%) What is the meaning of the *max-flow min-cut theorem*?
   (e). (3%) What is the importance of the *max-flow min-cut theorem*?

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5. The diameter of a graph $G=(V,E)$ is the greatest distance between any pair of vertices. (i.e. diameter = max { d(u, v) / for every pair of vertices u and v} ,where d(u, v) is the length of the shortest path between u and v)
   (a). (5%) Give a $O(|V|^3)$ algorithm (assume the graph is represented in the adjacency matrix form) to determine the diameter of graph G, where V is the set of vertices in G.
   (b). (5%) Prove the correctness of your algorithm.

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科目：演算法(B)　　　　日期：101 年 1 月 17 日　第 1 頁 共 2 頁

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1. 16%

(a)Use Dynamic programming technique to find an optimal solution for the 0-1 knapsack problem.
(Define the object function, write the recursive relation, give the initial condition, illustrate the table, indicate where the answer is, and estimate the time complexity and space complexity.)
(See the problem definition below. Assume that all the data, v_i w_i and W, are positive integers.)

(b)Is the algorithm a polynomial time algorithm? Explain briefly.

0-1 knapsack problem :
There are n items, the i th item is worth v_i dollars and weights w_i pounds for i=1 to n. A person wants to take as much valuable a load as possible but he can carry at most W pounds in his knapsack. Each item must either be taken or left behind. He cannot take a fraction of an item or take an item more than 0. Assume that all the data, v_i w_i and W, are positive integers.

2. Assume that the subset-sum problem and the set- partition problem are NP-complete, and assume $P \neq NP$.

(1)4% Determine whether each of the following two problems (A) and (B) is in P or is NP-hard

(2) 14% If it is in P, describe an algorithm briefly, and state the time complexity.
If it is NP-hard, prove your answer.

There are n programs and two storage devices (say two tapes or two disks). Let a_i be the length of the storage space needed to store the i th program, i= 1, 2, 3…,n. Let L_1 and L_2 be the length of the storage capacity of the storage devices respectively. (L_1 , and L_2 and a_i's are input variables.)

(A) Determine the maximum number of these n programs that can be stored on these two storage devices.

(B) Determine the maximum total length of these n programs that can be stored on these two storage devices.

Note: The rule here to store a program on the tape is the following: A program has to be stored consecutively as a whole into one tape. One program cannot be split into several small parts and stored in different tapes, nor can be stored only part of it.

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3. 16%　Assume that all the edge weights are positive.

(1) The longest-simple-cycle problem is the problem of determining a simple cycle (no repeated vertices) of maximum length in a graph.
Formulate it into a related decision problem.
Is the decision problem NP-complete? Or is it a P problem? Justify your answer.

(2) The smallest-simple-cycle problem is the problem of determining a simple cycle of minimum length in a graph.
Formulate it into a related decision problem.
Is the decision problem NP-complete? Or is it a P problem? Justify your answer.

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