

國立交通大學試題紙

科目：演算法 B

日期：109 年 7 月 15 日 第 1 頁 共 2 頁

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* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. 15% The subset sum problem is stated as follows:

Given a set $S = \{a_1, a_2, a_3, \dots, a_n\}$ of n positive integers and an integer target $t > 0$,determine whether there exists a subset S' of S whose elements sum to t .

(1) Use dynamic programming technique to solve the subset sum problem.

(Write the object function, recursive relation, initial condition, and the answer.

Give the time and space complexity of your algorithm.)

(2) Is your algorithm a polynomial time algorithm? Explain.

2. 10% Amortized analysis:

Consider the dynamic table problem (with both inserting and deleting operations). Doubling the table size when it is full. But instead of contracting the table by halving its size when its load factor drops below $1/4$, we contract it by multiplying its size by $2/3$ when its load factor drops below $1/3$. Define a (tight) potential function so that the amortized cost of each operation is bounded above by a constant (as small as possible).

(1) Write a tight potential function. (No need to explain.)

(2) Give the (smallest) constant that the amortized cost of each operation is bounded above by.
No need to prove it.

(3) Give the amortized cost for the following four cases. No need to explain.

The load factor is full 1, and the next operation is insertion.

The load factor is $1/3$, and the next operation is insertion.The load factor is $1/3$, and the next operation is deletion.The load factor is strictly between $1/3$ and $1/2$, and the next operation is deletion.

3. 15% Given a weighted, directed graph $G = (V, E)$ with no negative-weight cycles, let m be the maximum over all vertices $v \in V$ of the minimum number of edges in a shortest path from the source s to v . (Here the shortest path is by weight, not the number of edges).

(1) Describe briefly an algorithm to find a shortest path from the source s to v .(2) Explain briefly the time complexity in terms of m , V and E .

(3) Now consider weighted, directed graphs with negative-weight cycles.

Prove that the problem of finding a shortest path from the source to a destination is NP-hard.

◎請用深黑色鋼筆或原子筆出題

命題老師簽名：

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4. 10% CNF-satisfiability problem. (CNF= Product of Sum. DNF=Sum of product)

(1)Describe briefly that the DNF- satisfiability (search solution) problem has a polynomial time algorithm.

(2)It is known that every CNF Boolean expression can be rewritten as a DNF Boolean expression using Distributive law or DeMorgan's law. (For example, $(X+Y)Z=XZ+YZ$)

Consider the following algorithm to solve CNF- satisfiability (search solution) problem:

Given a CNF Boolean expression, we first transform it into a DNF Boolean expression using Distributive law, and then use the method in Part (1) to solve the DNF Boolean expression.

This will give a solution for the original CNF Boolean expression.

Is this a polynomial time algorithm to solve CNF Boolean expression? Explain.