

科目：計算理論 A

日期：102 年 1 月 30 日 第 1 頁 共 1 頁

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* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. (10%) Prove or disprove that if A is regular and $A \cap B$ is also regular, B is regular.

2. (10%) Prove or disprove that if A and B are both nonregular, $A \cap B$ is also nonregular.

3. (10%) Prove or disprove that the following language

$$A/B = \{w \mid wx \in A \text{ for some } x \in B\}$$

is regular if A is regular and B is context free.

4. (10%) Prove or disprove that the following language

$$A \diamond B = \{xy \mid x \in A \text{ and } y \in B \text{ and } |x| = |y|\}.$$

is regular if A and B are both regular.

5. (10%) Prove or disprove that the string 001101010 can be generated by grammar $G = \{\{S, A, B\}, \{0, 1\}, P, S\}$, where P contains the following productions:

$$\begin{aligned} S &\rightarrow 00B \\ A &\rightarrow 1B1 \mid \varepsilon \\ B &\rightarrow A0 \mid 0A \end{aligned}$$

科目：計算理論 B

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答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. (25%) Let $L = \{ \langle M \rangle : M \text{ is a Turing machine and } L(M) \text{ contains either string } aab \text{ or } bba \}$, where $\langle M \rangle$ is the standard encoding of Turing machine M .

(a) Show that L is not recursive by the reduction method.

(b) Show that L is recursively enumerable by giving the enumeration method.

2. (25%)

(a) What is the computational complexity class NP?

(b) What is the computational complexity class NP-complete?

Consider the **subset sum problem**: given a set $S = \{x_1, x_2, \dots, x_n\}$ and an integer T , determine whether there is subset $Y = \{y_1, y_2, \dots, y_k\} \subseteq S$ such that $\sum y_i = T$.

(c) Show that the following the language SUBSET-SUM is in NP.

$$\text{SUBSET-SUM} = \{ \langle x_1, x_2, \dots, x_n, T \rangle \mid \text{there is } Y = \{y_1, y_2, \dots, y_k\} \subseteq S, \sum y_i = T \}$$

(d) Show that SUBSET-SUM is NP-complete by a polynomial-time reduction from 3SAT.

* A recursive language is also called a "Turing-decidable" or simply "decidable" language.

* A recursively enumerable language is also called a "Turing-recognizable" language.