

科目：計算理論(A)

日期：100年1月26日 第1頁共1頁

請“✓”明 ✓不可看書 可看書

* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. (8%) Prove or disprove that the following language

$$A' = \{xz \mid (\exists y)(|x| = |y| = |z| \text{ and } xyz \in A)\}$$

is regular if A is regular.

2. (8%) Prove or disprove that the following language

$$A' = \{xz \mid (\exists y)(|x| = |y| = |z| \text{ and } xyz \in A)\}$$

is context free if A is context free.

3. (8%) Prove or disprove that the following language

$$A/B = \{w \mid wx \in A \text{ for some } x \in B\}$$

is regular if A and B are both regular.

4. (8%) Prove or disprove that the following language

$$A/B = \{w \mid wx \in A \text{ for some } x \in B\}$$

is context free if A and B are both context free.

5. (9%) Prove or disprove that the following language

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

is context free if A is context free and B is regular.

6. (9%) Prove or disprove that the following language

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

is context free if A is regular and B is context free.

科目：計算理論(B)

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7. (7%) Prove that if $A \leq_m B$ (i.e., language A is mapping reducible to language B) and A is undecidable, then B is undecidable.
8. (8%) Prove that if B is NP-complete and $B \leq_P C$ (i.e., A is mapping reducible to B in polynomial time) for C in NP, then C is NP-complete.
9. (15%) What is Rice Theorem? Prove it.
10. (20%) The Partition Problem is defined as follows. Given a set of n positive integers, $S = \{x_1, \dots, x_n\}$, does there exist a partition of S into two disjoint sets A and B such that $A \cup B = S$ and

$$\sum_{x_i \in A} x_i = \sum_{x_j \in B} x_j?$$

The Partition Problem is already proved to be NP-complete.

Now consider the following problem: Given a polygonal cycle with edges of integral lengths ℓ_1, \dots, ℓ_n and an integer L , can the cycle be folded flat (i.e., reconfigured so that the angle of each pair of adjacent edges is either 0 or 180°) so that its total folded length is at most L ? Note that each edge is a straight segment of integral length. For example, any triangle of integer edge lengths cannot be folded flat, while it is feasible for any square with L twice as the edge length. Prove that this problem is NP-complete by reducing the Partition Problem to it.