國 立 交 通 大 學 試 題 紙

九十六學年度第一次
博士班資格考

科目：計算理論

日期： 97 年 1 月 24 日 第 1 頁 共 1 頁

1. (30%) Let $\Sigma = \{0,1\}$, $w \in \Sigma^*$, w^R denote the reverse of w, and $\#_a(w)$ denote the number of symbol a in string w, where $a \in \Sigma$. Please prove or disprove the following statements:
   (a) (5%) $L_{1a} = \{w \mid \#_0(w) \text{ is even and } \#_1(w) \text{ is odd}\}$ is regular.
   (b) (5%) $L_{1b} = \{w \mid \#_0(w) = \#_1(w)\}$ is regular.
   (c) (10%) $L_{1c} = \{w \mid \#_0(w) = \#_1(w)\}$ is context free.
   (d) (10%) $L_{1d} = \{w \mid w = w^R \text{ and } \#_0(w) = \#_1(w)\}$ is context free.

2. (10%) If A is a set of natural numbers and k is a natural number greater than 1, let

$$B_k(A) = \{w \mid w \text{ is the representation in base } k \text{ of some number in } A\}\ .$$

   Here, we do not allow leading 0s in the representation of a number. For example, $B_2(\{3,5\}) = \{11, 101\}$ and $B_3(\{3,5\}) = \{10, 12\}$. Give an example of a set A for which $B_2(A)$ is regular but $B_3(A)$ is not regular. Prove that your example works.

3. (10%) If A and B are languages, define

$$A \diamond B = \{xy \mid x \in A \text{ and } y \in B \text{ and } |x| = |y|\}\ .$$

part A

   Show that if A and B are regular, then $A \diamond B$ is context free.

part B

4. (10%) Prove that a language is decidable if and only if it is Turing-recognizable (r.e.) and co-Turing-recognizable (co-r.e.).

5. (10%) Language A is *mapping reducible* to language B, written $A \leq_m B$, if there is a computable function $f : \Sigma^* \to \Sigma^*$, where for every w, $w \in A \iff f(w) \in B$. Prove that if $A \leq_m B$ and B is decidable, then A is decidable.

6. (10%) Define $L = \{< M, N > \mid M, N \text{ are Turing machines and } L(M) = L(N)\}$. Prove that L is neither Turing-recognizable nor co-Turing-recognizable.

7. (10%) A *2cnf-formula* is an AND of clauses, where each clause is an OR of at most two literals. Let $2SAT = \{< \phi > \mid \phi \text{ is a satisfiable 2cnf-formula}\}$. Show that $2SAT \in P$.

8. (10%) Consider the following problem: Let S be a set of integers. Can we partition S into two parts: S_1 and S_2 such that the sum of integers in S_1 is equal to the sum of integers in S_2? Prove that this problem is NP-complete. You can use the problem $SUBSET - SUM$ for reduction.

◎請用深黑色鋼筆或原子筆出題

命題老師簽名：