國 立 交 通 大 學 試 題 紙

一百零六學年度第二次
博士班資格考

科目:演算法 A

日期: 107 年 7 月 31 日 第 1 頁 共 1 頁

請"✓"明 ✓不可看書 可看書

* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. Briefly describe the algorithm to delete a specific element from a Fibonacci Heap. *11%*
2. Minimum Spanning Tree: Describe the Kruskal's algorithm and the Prim's algorithm. What is the data structure each algorithm requires. If the graph has n vertices and m edges, what is the time complexity for each of the algorithms. *12%*
3. Suppose that you have a computational problem A, input to A can be transformed to the input to sorting problem in linear time. We solve the sorting problem, and the solution to the sorting problem is transformed back to the solution to the problem A (so we solve problem A) in linear time. Since the lower bound to sorting problem is $\Omega(n \log n)$, lower bound to A is also $\Omega(n \log n)$. Is this argument correct? Give me your reason(s). *15%*
4. Given recursion $T(n) = T(n/5) + T(3n/4) + n$ with boundary condition $T(1)=O(1)$, argue that $T(n)$ is bounded above by $O(n)$. *12%*

◎ 請用深黑色鋼筆或原子筆出題

命題老師簽名：

國 立 交 通 大 學 試 題 紙

一百零六學年度第二學期
博士班資格考

科目:演算法 B

日期: 107 年 7 月 31 日 第 1 頁 共 1 頁

請"✓"明 ✓不可看書 可看書

* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. 14% For each of the following two problems decide whether it is a P-problem or an NP-hard problem. If it is P, name a polynomial time algorithm to solve it and justify your answer briefly. If it is NP-hard, show it.
   (1) Find a negative-weight cycle in a weighted directed graph G=(V, E).
   (2) Find, among all negative-weight cycles, one that contains the most number of edges.

2. 12% Let $G=(V, E)$ be a directed graph with positive edge weight.
   The diameter of a graph $G=(V, E)$ is the maximum of the distances between any pair of vertices.
   (i.e. diameter = max $\{d(u, v)$ / for every pair of vertices *u and v*$\}$,
   where $d(u, v)$ is the length of the shortest path between u and v)
   Determine whether the problem of finding the diameter of a graph with positive weight is in P or is NP- hard? If it is NP-hard, prove it. If it is P, describe an algorithm briefly.

3. 12% For the following two problems, decide whether it is a P-problem or an NP-hard problem.
   If it is P, explain briefly. If it is NP-hard, show it.
   (You may use some special terms to explain the idea, no need to write the details)
   (1) Find a minimum cut in a flow network G.
   (2) Find, among all minimum cuts in a flow network G, one that contains the smallest number of edges.

4. 12% Assume the basic knowledge of NP-completeness: satisfiability, 3-CNF, hamiltonian- cycle, vertex-cover, subset-sum and the partition problems are NP-complete. And assume $P \neq NP$
   Prove that the 0-1 integer linear-programming problem is NP-complete.
   You may use an example to explain the idea.
   (Given an integer $m \times n$ matrix A and an integer m-vector b, the 0-1 integer linear-programming problem asks whether there exists an n-vector x with element in the set $\{0, 1\}$ such that $Ax \leq b$.)

◎ 請用深黑色鋼筆或原子筆出題

命題老師簽名：