

國立交通大學試題紙

科目：演算法 A

日期：108 年 7 月 31 日 第 1 頁 共 1 頁

請 “✓” 明 ✓不可看書 可看書

* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. (10%) Show that building a max-heap takes linear time. So you have to present the algorithm (pseudo code if possible), then show your algorithm takes linear time.
2. (10%) Show that LSD Radix sort sorts n integer of k digits in linear time where k is a constant. So you need to present the linear time algorithm, then show the algorithm takes $O(n)$ time. Using Linear Decision Tree model, we can show that any sorting algorithm requires $\Omega(n \log n)$ time; why LSD beats the lower bound?
3. (10%) About binary search tree:
 - (a) Show that there are $n+1$ unused link fields if there are n nodes in the binary search tree. (When you implement a binary search tree, a node has a data field and two link fields, *leftChild* and *rightChild*. Link field is unused if it does not point to a node.)
 - (b) Describe the algorithm to delete a node from a binary search tree.
4. (10%) The hiring problem, I want to hire the best one from n candidates. The candidates come in random order, I interview one (cost me c_i), if this one is better than the one I have hired, then I hire the new one to replace the old one (cost me c_h). : the cost for an interviewing c_i , usually a small cost; the cost for hiring c_h , usually a much more expensive one; total cost is thus $O(c_i \cdot n + c_h \cdot m)$ where m is the number of times we hire a new one to replace the previous one. What is the best case for m ? What is the worst case for m ? Show that m is $O(\ln n)$ in the average. You may need the equation

$$\sum_{i=1}^n \frac{1}{i} = \ln n.$$

5. (10%) Implement a queue using two stacks:
 - (a) Describe the algorithm `InQueue(a)`, use pseudo code.
 - (b) Describe the algorithm `DeQueue()`, use pseudo code.
 - (c) Use potential method to show that both `InQueue(a)` and `DeQueue()` take $O(1)$ amortized time.

◎請用深黑色鋼筆或原子筆出題

命題老師簽名：

國立交通大學試題紙

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1. 13%

(1) Use dynamic programming technique to solve the all-pairs shortest paths problem in a directed graph. (So called Floyd-Warshall algorithm)

(Define the object function; state the recursive formula, initial condition, and answer. Give the time complexity and space complexity without explanation.)

(2) Suppose that some of the edge weights are negative, what can you say about the algorithm given in part (1)? Do you still get the all-pairs shortest simple path correct? How do you detect the existence of a negative weight directed cycle? How do you find one such cycle?

2. 13%

Consider the CNF-satisfiability problem.

(CNF= Product of Sum. DNF=Sum of product)

(1) Suppose someone has a polynomial time algorithm to solve CNF-satisfiability (decision) problem, show that you can use it to obtain a polynomial time algorithm to solve CNF-satisfiability (search solution) problem.

(2) Prove briefly that the DNF- satisfiability (search solution) problem has a polynomial time algorithm and state the time complexity roughly.

(3) It is known that every CNF Boolean expression can be rewritten as a DNF Boolean expression using Distributive law or DeMorgan's law. (For example, $(X+Y)Z=XZ+YZ$) Consider the following algorithm to solve CNF- satisfiability (search solution) problem:

Given a CNF Boolean expression, we first transform it into a DNF Boolean expression, then use the method in Part (2) to solve the DNF Boolean expression.

This will give a solution for the original CNF Boolean expression.

Is this a polynomial time algorithm to solve CNF Boolean expression? Why?

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3. 12%

Use the fact that finding a Hamiltonian path or Hamiltonian cycle (in directed graphs or undirected graph) is NP-hard;

- (1) prove that the problem of finding a shortest simple path between two nodes in a directed graph with negative weight directed cycle is NP-hard.
- (2) Assume that there are NO negative weight directed cycles. Is the problem of finding a shortest simple path between two nodes P or NP-hard ? Explain it briefly.

4. 12%

- (1) Use dynamic programming technique to solve the optimal binary search tree problem. (Firstly, state the notations and define the object function. Then write the recursive formula and initial condition. Where is the final solution? Estimate the time complexity and the space complexity.) You may write it in an informal way.

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