

科目：生物統計

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請“✓”明 ✓不可看書 可看書

* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. (50 points) Suppose that X is a discrete random variable taking three possible values (1,2,3) with the following probability function:

| Value of X | 1 | 2 | 3 |
|--------------|---------------|---------------|---------------|
| $\Pr(X = x)$ | $\frac{1}{6}$ | $\frac{2}{6}$ | $\frac{3}{6}$ |

Table 1-1: probability function of X

a. Please compute the values of $E(X)$ and $Var(X)$

Suppose that you write computer codes to generate a random sample of X , denoted as X_1, \dots, X_{100} .

b. The Central Limit Theorem tells you the distribution of $\bar{X} = \sum_{i=1}^{100} X_i / 100$ is approximately normally distributed. Please use this theorem to find the approximate value of $\Pr(1.5 \leq \bar{X} \leq 2.5)$.

Note: The normal table is attached.

The data based on observations of X_1, \dots, X_{100} can be summarized as below:

| x | 1 | 2 | 3 | total |
|---------------------------------|----|----|----|-------|
| number of $X_i = x$ | 15 | 25 | 60 | 100 |
| Expected number of $X_i = x$ | | | | |

Table 1-2: Summary of data generated from X

c. Please obtain the sample estimate of $E(X)$.

d. Please use the Pearson Chi-squared test to verify that your codes are written correctly or not. The level of significance is 5%. Hint: first compute the expected values in Table 1-2.

Note: The Chi-squared table is attached.

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2. (50 points) Two brands of laundry detergent ("Best" and "Supper") are compared based on their performances on removing the amount of dirt from the laundry.

The raw data for "Best"(sample 1) are 4, 3, 5, 5, 7, 5

The raw data for "Supper"(sample 2) are 7, 3, 6, 4, 5, 8

The sample mean and standard deviation for each brand are $\bar{X}_1 = 4.83, S_1 = 1.33$; $\bar{X}_2 = 5.5, S_2 = 1.87$, respectively.

a. (5 points) Please estimate the standard deviation of $\bar{X}_1 - \bar{X}_2$.

b. (15 points) Please test $H_0: \mu_1 = \mu_2$ versus $H_a: \mu_1 \neq \mu_2$ at 5% level of significance. (For conservative purposes, set the degree of freedom as $\min(n_1 - 1, n_2 - 1) = 5$ for the two-sample t test.)

Compute the range of p-value and then state your conclusion whether $H_0: \mu_1 = \mu_2$ is rejected or not.

Note: The T table is attached.

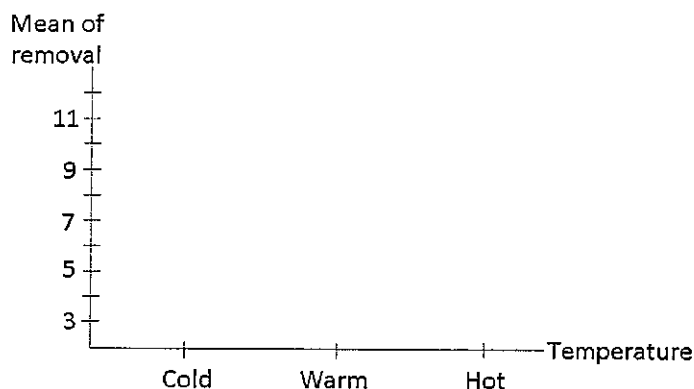
The above experiment was run based on "cold" water. Additional experiments were designed by adding two more levels (namely "warm" and "hot"). The whole data are given below.

| | Cold | Warm | Hot |
|--------|--------------------------------------|--|--------------------------------------|
| Best | 4, 3, 5, 5, 7, 5 cell mean = 4.83 | 9, 8, 5, 10, 11 cell mean = 8.6 | 9, 12, 14, 9, 10 cell mean = 10.8 |
| Supper | 7, 3, 6, 4, 5, 8 cell mean = 5.5 | 15, 12, 16, 14, 15 cell mean = 14.4 | 9, 10, 15, 11, 10 cell mean = 11 |

Table 2-1: Data based on the two-way design

c. (10 points) Please draw the 6 cell mean values on the plot and then give your comments.

Note: Please draw the plot by yourself on the answer sheet and then mark the mean values.



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Problem 2: Continued

d. (15 points) Formally we can analyze the data in Table 2-1 using two-way ANOVA with interaction. Based on the results in Table 2-2, test whether the two main effects ("brand" and "temperature") and the interaction effect exist or not. Since the computer output already provides the p-values, F table is NOT attached.

| | Df | Sum Sq | Mean Sq | F value | Pr(>F) | |
|------------|----|--------|---------|---------|----------|-----|
| Temp | 2 | 274.81 | 137.40 | 36.629 | 2.73e-08 | *** |
| Brand | 1 | 36.12 | 36.12 | 9.630 | 0.00457 | ** |
| Temp:Brand | 2 | 49.41 | 24.70 | 6.586 | 0.00485 | ** |
| Residuals | 26 | 97.53 | 3.75 | | | |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Table 2-2: Output based on Two-way ANOVA with interaction

e. (5 points) Please state why the plot in (c) does not provide enough evidence to compare the effects.