國立交通大學試題紙

一百零五學年度第二次
博士班資格考

科目:演算法 A 日期: 106 年 7 月 27 日 第 1 頁 共 1 頁

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1. Selection Problem: Find the k-th largest in an array of size n.

   (a) In a special case, we are looking for the largest (so $k = n$). It is obvious that the largest can be found in $O(n)$ time. Do you think it needs $\Omega(n)$ time? Why? 5%

   (b) If k is the median, if instead of partitioning them into 5 in a group, I have 10 in a group. Describe the algorithm to find the median. Please also give me the recursion if I insist partition them into 10 in a group. What is the time complexity if I do so. 12%

2. Sorting:

   (a) Selection problem (the previous one) can be solved in linear time. To build a max heap can be done in linear time as well. To sort, however, it needs at least $\Omega(n \log n)$ time to sort n numbers? Give me your comments about this. 5%

   (b) Show that the height of a binary tree is $\Omega(\log n)$ if there are n nodes in the tree. 8%

   (c) Given a max-heap of n nodes, prove by induction that the left child and right child of node i are respectively $2i$ and $2i + 1$ if $(2i + 1) \leq n$. Note that the max-heap is a complete binary tree stored in an array. Node i means the nodes stored in index i in the array. We don't used index 0. 8%

3. Dynamic table: C++ STL (standard template library) support vector. You use that as an array (support direct access, i.e., to access `A[i]` in constant time) but the nice thing is that you don't need to specify the size. If you make a sequence of n insertions and deletions, STL guarantees the space of the vector is $O(n)$, and the cost to management the space is $O(n)$ as well. Describe how to do the space management and show that the amortized cost is linear. Note that, there are table expansion and contraction. 12%

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命題老師簽名:

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1. The subset sum problem is stated as follows:

   Given a set $S = \{ a_1, a_2, a_3, \dots, a_n \}$ of n positive integers and an integer target $t > 0$, determine whether there exists a subset S' of S whose elements sum to t.

   (1) Use dynamic programming technique to solve the subset sum problem.
   (Write the object function, recursive relation, initial condition, and illustrate the table, indicate where the answer is. Give the time and space complexity of your algorithm.)

   (2) Apply your algorithm to solve the example $S= \{2, 1, 3\}$ and $t= 4$. Show the table.

   (3) Is the algorithm a polynomial time algorithm? Explain.

15%

2. Let $G= (V, E)$ be a directed graph with positive edge weight.
   The diameter of a graph $G= (V, E)$ is the maximum of the distances between any pair of vertices.
   (i.e. diameter = max {d(u, v) / for every pair of vertices *u and v*},
   where d(u, v) is the length of the shortest path between u and v)
   Determine whether the problem of finding the diameter of a graph with positive weight is in P or is NP- hard? If it is NP-hard, prove it. If it is P, describe an algorithm.

20%

3. Let $G= (V, E)$ be a graph with positive edge weight.

   (1) The longest-simple-cycle problem is the problem of determining a simple cycle (no repeated vertices) of maximum length in a graph.
   Formulate it into a related decision problem.
   Is the decision problem NP-complete? Or is it a P problem? Justify your answer.

   (2) The smallest-simple-cycle problem is the problem of determining a simple cycle of minimum length in a graph.
   Formulate it into a related decision problem.
   Is the decision problem NP-complete? Or is it a P problem? Justify your answer.

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