

國立交通大學試題紙

一百零四學年度第一次
博士班資格考

科目：演算法 A

日期：105 年 7 月 28 日 第 1 頁 共 2 頁

請“✓”明 ✓不可看書 可看書

* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. Consider the Strassen's matrix multiplication algorithm(a divide and conquer approach).
 - (1) Using traditional matrix product method, how many multiplications and additions do we need to multiply two 2×2 matrices? How about Strassen's method?
 - (2) Write the related recurrence equation, and give the asymptotic tight bound solution. (No need to give any explanation)
 - (3) Suppose that we use a similar divide and conquer approach to compute this problem; divide $n \times n$ matrices into $\frac{n}{2} \times \frac{n}{2}$ submatrices the same way as Strassen's, while in the "merge" part, there are m submatrix multiplications and a submatrix additions. Find the time complexity of this method.
 - (4) Suppose that we divide the matrices into $\frac{n}{3} \times \frac{n}{3}$ submatrices (and there are m submatrix multiplications and a submatrix additions in the "merge" part). Answer the same question as in part (3).
 - (5) For part (4), what is the largest value of m (submatrix multiplication) to get an asymptotic faster matrix multiplication algorithm than Strassen's ?
2. Consider the shortest path problem. (In a directed graph, find a shortest simple path between two given nodes s and t . Some of the edges may have negative weights.)
 - (1) Describe the Bellman-Ford algorithm. (No need to prove the correctness.)
 - (2) State and explain briefly the time complexity.
 - (3) Suppose that there exist negative weight directed cycles. How can we detect one such cycle. What does Bellman-Ford algorithm do?
 - (4) Suppose that all the edges are with positive weight. What is the most efficient algorithm to find a shortest path (Give the name of the algorithm, no need to describe it). State the time complexity. Give an example to show that this algorithm does not work, if there exist negative weight edges.
3. Consider the CNF-satisfiability problem.
(CNF= Product of Sum. DNF=Sum of product)

◎請用深黑色鋼筆或原子筆出題

命題老師簽名：

科目：演算法 A

日期：105 年 7 月 28 日 第 2 頁 共 2 頁

- (1) Suppose someone has a polynomial time algorithm to solve CNF-satisfiability (decision) problem, show that you can use it to obtain a polynomial time algorithm to solve CNF-satisfiability (search solution) problem.
- (2) Explain briefly that the DNF- satisfiability (search solution) problem has a polynomial time algorithm and state the time complexity roughly.
- (3) It is known that every CNF Boolean expression can be rewritten as a DNF Boolean expression. (Use Distributive law or DeMorgan's law. For example, $(X+Y)Z=XZ+YZ$) Consider the following algorithm to solve CNF- satisfiability (search solution) problem:
Given a CNF Boolean expression, we first transform it into a DNF Boolean expression, then use the method in Part (2) to solve the DNF Boolean expression.
This will give a solution for the original CNF Boolean expression.
Is this a polynomial time algorithm to solve CNF Boolean expression? Why?

◎請用深黑色鋼筆或原子筆出題

命題老師簽名：

國立交通大學試題紙

一百零四學年度第一次
博士班資格考

科目：演算法 B

日期：105 年 7 月 28 日 第 1 頁 共 1 頁

請“✓”明 ✓不可看書 可看書

* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. About the best algorithm to build a max heap. Describe the algorithm and show that the time required is linear.
2. Assume that we sort n integers of length k . Describe the linear time algorithm to sort these n integers. Show that the algorithm correctly sorts these n integers in linear time. It has been shown that under the linear decision tree model, any sorting algorithm takes time at least $(n \log n)$ time, but now you just got a linear time sorting algorithm, what went wrong?
3. The longest common subsequence can be found using dynamic programming. Try to show that the greedy algorithm doesn't work (cannot correctly find the LCS).

◎請用深黑色鋼筆或原子筆出題

命題老師簽名：