國立交通大學試題紙

一百學年度第二次
博士班資格考

科目：演算法(A)　　　　日期：101 年 7 月 26 日　第 1 頁 共 3 頁

請"✓"明　✓不可看書　可看書

* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. 16% Interval graphs, activity-selection problem.

Suppose we have a set $S = \{a_1, a_2, \dots, a_n\}$ of n proposed activities. Each activity a_i has a start time s_i and a finish time f_i, where $0 \le s_i < f_i < \infty$. If selected, activity a_i takes place during the half-open time interval $[s_i, f_i)$. Activities a_i and a_j are compatible if the intervals $[s_i, f_i)$ and $[s_j, f_j)$ do not overlap. That is, a_i and a_j are compatible if $s_i \ge f_j$ or $s_j \ge f_i$. In the activity-selection problem, we wish to select a maximum-size subset of mutually compatible activities.

For the following greedy idea, pick out the ones that yield an optimal solution. You must give proof.

(1) Selecting the first activity to start (that is compatible with all previously selected activities).

(2) Selecting the first activity to finish(that is compatible with all previously selected activities).

(3) Selecting the last activity to start (that is compatible with all previously selected activities).

(4) Selecting the last activity to finish (that is compatible with all previously selected activities).

(5) Selecting the activity of least duration (that is compatible with all previously selected activities).

(6) Selecting the compatible activity that overlaps the fewest other remaining activities.

◎請用深黑色鋼筆或原子筆出題

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2. 17% Minimum spanning trees.

In this problem, we give pseudocode for three different algorithms. Each one takes a graph as input and returns a set of edges T. For each algorithm, you must either prove that T is a minimum spanning tree or prove that T is not a minimum spanning tree.

a、 MAYBE-MST-A(G, w)

```
sort the edge into nonincreasing order of edge weights w
T←E
for each edge e, taken in nonincreasing order by weight
    do if T←{e} is a connected graph
        then T←T−e
return T
```

b、 MAYBE-MST-B(G, w)

```
T← ∅
for each edge e, taken in arbitrary order
    do if T∪{e}
        then T←T∪e
return T
```

c、 MAYBE-MST-C(G, w)

```
T← ∅
for each edge e, taken in arbitrary order
    do T←T∪e
        if T has a cycle C
            then let e' be the maximum-weight edge on c
                T←T−{e'}
return T
```

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3.

(A) 4% Let G be a directed graph with any kind of edge weights, positive and/or negative weights, having positive weight cycles and/or negative weight cycles. Assumes NP≠P. For each of the following problems classify whether it is NP-hard or P.

(1) Find a negative weight cycle.

(2) Find a negative weight cycle with the least absolute value total weight.

(3) Find a positive weight cycle.

(4) Find a positive weight cycle with the least total weight.

(B) 13% Prove your answers. (You may explain briefly, give a simple illustration, or give the name of the algorithm.)

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科目：演算法(B)　　　　日期：101 年 7 月 26 日　第 1 頁 共 1 頁

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答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. (15%) Rank the following functions by order of growth; that is, find an arrangement $g_1, g_2, \ldots, g_n$, of functions satisfying $g_1 = \Omega(g_2)$, $g_2 = \Omega(g_3)$, $\ldots$, $g_{n-1} = \Omega(g_n)$. Partition your list into equivalence classes such that $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$. Hint: derive the simplest formula for each function.
   - (a) $(\log n)^{\log 2n}$
   - (b) $n(\log n) / (\log\log n)^3$
   - (c) $n^{\log (\log n)^2}$
   - (d) $\log((n \log n)^{n \log n})$
   - (e) n^n
   - (f) $(1) + (1 + 1/2) + (1 + 1/2 + 1/3) + \ldots + (1 + 1/2 + \ldots + 1/n)$
   - (g) 2^{3^n}
   - (h) 3^{2^n}
   - (i) $(n!)!$
   - (j) $(n!)^n$
   - (k) $1\times(n-1)+2\times(n-2)+ \ldots + (n-1)\times 1$
   - (l) $n + n/2^2 + n/3^2 \ldots + n/(n^2)$
   - (m) $2\times(n^2-2)+4\times(n^2-4)+ \ldots + (n^2-2)\times 2$
   - (n) $1 + 1/2 + 1/3 \ldots + 1/(10^n)$
   - (o) $2^{\lg n}$
   - (p) $4^{\lg n}$
2. (10%) In a hash table in which collisions are resolved by chaining, let the load faction be $\alpha = n/m$. Prove that an unsuccessful search takes expected time $\Theta(1 + \alpha)$, under the assumption of simple uniform hashing.
3. (13%) Given n integers $(a_1, a_2, \ldots, a_n)$, where the number of distinct integers is $O(\log n)$, design a comparison sort algorithm using at most $O(n \log\log n)$ comparisons. But, our textbook seems to say "the lower bound for sorting is $\Omega(n \log n)$" (this seems contradictory to your algorithm). Explain why.
4. (12%) Briefly describe the dynamic programming algorithm for optimal parenthesization of a matrix-chain product with the smallest number of scalar multiplications. Illustrate your algorithm for the matrix-chain product whose sequence of dimensions is {6, 7, 3, 1, 2, 4, 5}.

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