

科目：計算理論(A)

日期：99年7月28日 第1頁共1頁

請“✓”明 ✓不可看書 可看書

* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. (40%) Let $\Sigma = \{0,1\}$, \oplus stand for the bitwise XOR operator, and $\#_a(w)$ denote the number of symbol a in string w , where $a \in \Sigma$. Prove or disprove the following statements:

- (a) (10%) If A is a regular language over Σ , then

$$B(A) = \{0^{\#_0(x)}1^{\#_1(x)} \mid x \in A\},$$

is also regular.

- (b) (10%) If A and B are regular languages over Σ , then

$$A \oplus B = \{x \oplus y \mid x \in A, y \in B, |x| = |y|\},$$

is also regular.

- (c) (10%) For $x, w \in \Sigma^*$,

$$A = \{x \mid x \neq ww\},$$

is context-free.

- (d) (10%) For $x, w \in \Sigma^*$,

$$A = \{x \mid x = ww\},$$

is context-free.

2. (10%) Convert context-free grammar $G = \{\{S, A, B, C\}, \{a, b\}, P, S\}$, where P contains the following productions:

$$S \rightarrow aAa \mid bBb$$

$$A \rightarrow B \mid a$$

$$B \rightarrow C \mid b$$

$$C \rightarrow S \mid \varepsilon$$

into Chomsky normal form. Describe in detail the resultant grammar after each step.

科目：計算理論(B)

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3. (5%) Prove that a language A is decidable if and only if $A \leq_m 0^*1^*$.
4. (15%) Let $L = \{\langle G \rangle \mid G \text{ is an ambiguous CFG}\}$. Show that L is undecidable.
5. (15%) Prove that if $P=NP$, a polynomial algorithm exists that takes an undirected graph as input and finds a largest clique contained in that graph. State and explain your method to find the largest clique.
6. (15%) Let ϕ be a 3CNF-formula. We say ϕ is NAE-satisfiable if there is a truth assignment such that ϕ is satisfied and in each clause of ϕ not all literals are set to be true. Define $\text{NAE-SAT} = \{\phi \mid \phi \text{ is NAE-satisfiable}\}$. Show how to use the following reduction from 3SAT to NAE-SAT to prove that NAE-SAT is NP-complete: replace each clause $c_i = (y_1 \vee y_2 \vee y_3)$ with two clauses $(y_1 \vee y_2 \vee z_i)$ and $(\bar{z}_i \vee y_3 \vee b)$, where z_i is a new variable for each clause c_i and b is a single additional new variable.