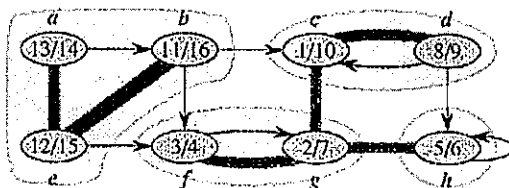


科目：演算法 A

日期：104 年 1 月 27 日 第 2 頁 共 2 頁

3. (10%) For a directed graph, design an algorithm to find strongly connected components or SCCs (each shaded area is a SCC in the following graph) and explain the correctness of this algorithm by illustrating the following graph.



4. (10%) A *bottleneck spanning tree* T of an undirected graph G is a spanning tree of G whose largest edge weight is minimum over all spanning trees of G . We say that the value of the bottleneck spanning tree is the weight of the maximum-weight edge in T . Argue that a minimum spanning tree is a bottleneck spanning tree.

◎ 請用深黑色鋼筆或原子筆出題

命題老師簽名：

科目：演算法 A

日期：104 年 1 月 27 日 第 1 頁 共 2 頁

請“✓”明 ✓不可看書 可看書

* 請將答案依題號順序寫入答案卷

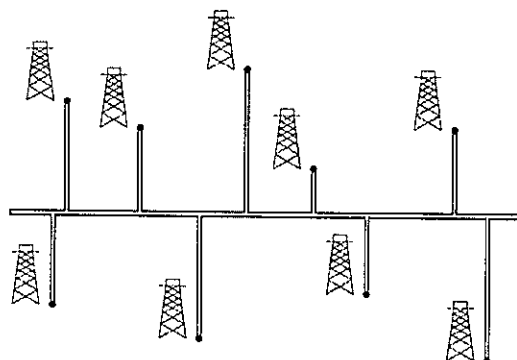
答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. (20%) Rank the following functions by order of growth; that is, find an arrangement g_1, g_2, \dots, g_n , of functions satisfying $g_1 = \Omega(g_2)$, $g_2 = \Omega(g_3)$, \dots , $g_{n-1} = \Omega(g_n)$. Partition your list into equivalence classes such that $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$.

Hint: $\log n = \log_{10} n$, $\lg n = \lg_2 n$, $\ln n = \log_e n$, $\log_e 2 = 0.693$ and $\log_{10} 2 = 0.301$.Assume that $f(n)$ is constant for sufficiently small n .

- | | |
|---|---|
| (a) $(\lg n)^{2 \lg n}$ | (b) $n (\lg n) / (\lg \lg n)^3$ |
| (c) $(2n)^{1/\lg n}$ | (d) $(2n)^{\lg \lg n}$ |
| (e) 1.1^n | (f) 1.01^{10n} |
| (g) $2n \times (2n-2) \times \dots \times 2$ | (h) n^n |
| (i) $\log((n \log n)!)$ | (j) $\log((n^2)!)$ |
| (k) $8^{\lg n}$ | (l) $n!$ |
| (m) $n^2 + n^2/2 + n^2/4 + \dots + 1$ | (n) $1^3 n + 2^3(n/2) + 3^3(n/4) + \dots + (\ln n)^3$ |
| (o) $1/1 + (1+1/2) + (1+1/2+1/3) + \dots + (1+1/2+1/3+\dots+1/n)$ | |
| (p) $1 \times (n-1) + 2 \times (n-2) + 3 \times (n-3) + \dots + (n-1) \times 1$ | |
| (q) $n^2 + n^2/2^2 + n^2/3^2 \dots + n^2/n^2$ | (r) $f(n) = 2f(n/2 + 17) + n$. |
| (s) $f(n) = 2f(n/2) + n \log n$ | (t) $f(n) = 2f(n/2) + n / \log n$. |

2. (10%) An oil company is planning a large pipeline running east to west through an oil field of n wells. From each well, a spur pipeline is to be connected directly to the main pipeline along a shortest path (either north or south), as shown below. Given coordinates (x, y) of the wells, how should the company pick the optimal location of the main pipeline (the one that optimizes the total length of the spurs)? Show the optimal location can be determined in linear time.



◎ 請用深黑色鋼筆或原子筆出題

命題老師簽名：

科目：演算法 B

日期：104 年 1 月 27 日 第 1 頁 共 1 頁

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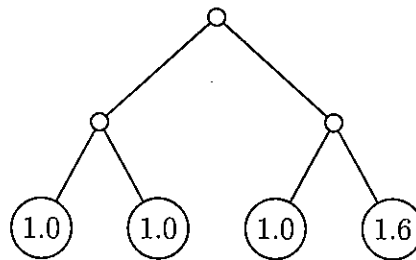
* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

- (15%) For any k positive real numbers x_1, \dots, x_k , from the open interval $(0, 1)$, find a number s such that $x_1^s + \dots + x_k^s = 1$. You can assume with precision after t decimal places, where t is a fixed integer. First prove that the s always exists and is unique. Then give an algorithm to find s with precision after t decimal places, and prove the complexity.
- (15%) In this problem we want to build a full binary tree (i.e., each internal node has 2 children) to store N values, s_1, \dots, s_N , in the leaf nodes. Let the tree root be at the 0-th level. For some full binary tree, suppose s_j is stored at the ℓ_j -th level of the binary tree. We are interested in the following value:

$$\max_{j=1, \dots, N} \{s_j (\frac{1}{2})^{\ell_j}\}.$$

For example, suppose we have 4 s -values: 1.0, 1.0, 1.0 and 1.6. Then the following binary tree yields the maximum number $\max_j \{s_j (\frac{1}{2})^{\ell_j}\} = 0.4$.



Design and analyze an algorithm to construct a full binary tree such that $\max_j \{s_j (\frac{1}{2})^{\ell_j}\}$ is minimized.

- (10%) In a coordinate plane starting at the origin point $(0, 0)$, there are three possible moves: L -move, R -move and U -move. Let (x, y) be the current position. Then $L(x, y) = (x - 1, y)$ indicates moving left by a unit. Similarly, $R(x, y) = (x + 1, y)$ and $U(x, y) = (x, y + 1)$. However, L -move cannot be followed immediately by an R -move, and vice versa. Give an algorithm to find the number of possible paths of length N with the above three moves.
- (10%) Make a polynomial time reduction from the 3SAT problem to the Vertex Cover problem.

◎ 請用深黑色鋼筆或原子筆出題

命題老師簽名：