國立交通大學試題紙

一百零三學年度第二次
博士班資格考

科目：演算法 A　　日期：104 年 7 月 30 日　第 1 頁 共 1 頁

請" ✓" 明　✓不可看書　可看書

* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

Algorithms:

1. (10 %) Let A and B be two sets, each containing n positive integers. You can order both set in any way. Let a_i be the i-th element of A, and let b_i be the i-th element of B. Give an algorithm that will maximize $\Pi_{i=1}^{n} a_i^{b_i} := a_1^{b_1} \times a_2^{b_2} \times \cdots \times a_n^{b_n}$. Explain and analyze the complexity your answer.

2. (10%) Suppose we are given a set $S = \{p_1, p_2, \ldots, p_n\}$. For each pair p_i and p_j, there is a distance $d(p_i, p_j) = d(p_j, p_i) > 0$, for $i \neq j$. We want to partition S into k groups: $\{C_1, \ldots, C_k\}$, which is called the k-clustering. The *measure* of a k-clustering is the minimum distance between any pair of points lying in different groups. We want to find a k-clustering such that the *measure* is maximum. Give an efficient algorithm to find the maximum measure. Explain and analyze the complexity your answer.

3. (10%) Consider n distinct positive integers $x_1, \ldots, x_n$. Give an efficient algorithm to find an integer y such that $\sum_{i=1}^{n} |x_i - y|$ is minimal. Explain and analyze the complexity your answer.

4. Consider the following 10 clauses as an instance of 2SAT (where each clause contains at most 2 literals):

$$(x), (y), (z), (w)$$
$$(\bar{x} \vee \bar{y}), (\bar{y} \vee \bar{z}), (\bar{z} \vee \bar{x})$$
$$(x \vee \bar{w}), (y \vee \bar{w}), (z \vee \bar{w})$$

   (a) (10%) Show that if $(x \vee y \vee z)$ is satisfied, then 7 of the above clauses are satisfied and no more; otherwise at most 6 clauses can be satisfied.

   (b) (10%) Given a positive integer K and an instance ϕ of 2SAT, we want to find a truth assignment such that at least K clauses of ϕ are satisfied. This is the so called MAX2SAT problem. Using (a), give a polynomial time reduction from 3SAT to MAX2SAT.

◎ 請用深黑色鋼筆或原子筆出題

命題老師簽名：

國立交通大學試題紙

一百零三學年度第二次
博士班資格考

科目:演算法 B　　　　日期： 104 年 7 月 30 日　第 1 頁 共 2 頁

請" ✓" 明　　✓不可看書　　可看書

* 請將答案依題號順序寫入答案卷

答題時字跡需工整，否則不予計分。Write your answers legibly; otherwise you will get zero score.

1. 17%

Consider a two-position switch with two inputs and two outputs, as shown in Fig. A. In one position inputs 1 and 2 are connected to outputs 1 and 2, respectively. In the other position inputs 1 and 2 are connected to outputs 2 and 1, respectively. Using these switches and applying the divide and conquer technique, we can design a network with n inputs and n outputs which is capable of achieving any of the n! possible permutations of the inputs, as shown in Fig. B.

(1) Draw the switch network completely for n=8..

(2) For general $n=2^k$, find the number of switches used in the networks. (Write your answer in terms of n and/or k. Give the exact number, NOT just the asymptotic big O order.)

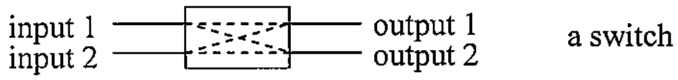

Fig.A

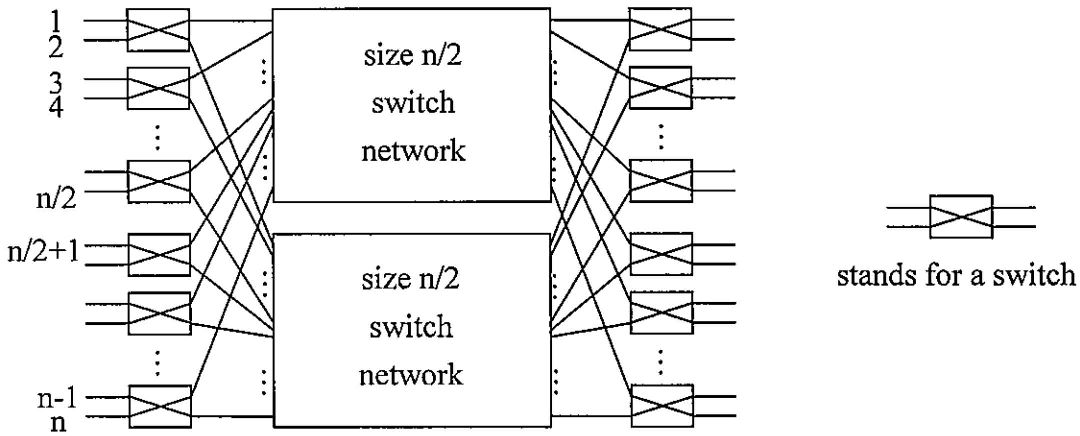

Fig.B

2 16%

Use dynamic programming technique to find the longest monotonically increasing subsequence of a sequence of n numbers.

(Write the object function, recursive relation, initial condition, and the answer. Give the time and space complexity of your algorithm.)

◎請用深黑色鋼筆或原子筆出題

命題老師簽名：

國立交通大學試題紙

科目:演算法 B

日期： 104 年 7 月 30 日

3 17%

The diameter of a graph $G=(V,E)$ is the maximum of the distances between any pair of vertices. (i.e. diameter = max { d(u, v) / for every pair of vertices u *and* v },
where d(u, v) is the length of the shortest path between u and v.)

Let $G=(V,E)$ be a weighted directed graph. Each edge (u v) has a positive weight w(u, v).

Give a $O(|V|^3)$ algorithm to determine the diameter of graph G, where V is the set of vertices in G.

◎請用深黑色鋼筆或原子筆出題

命題老師簽名：